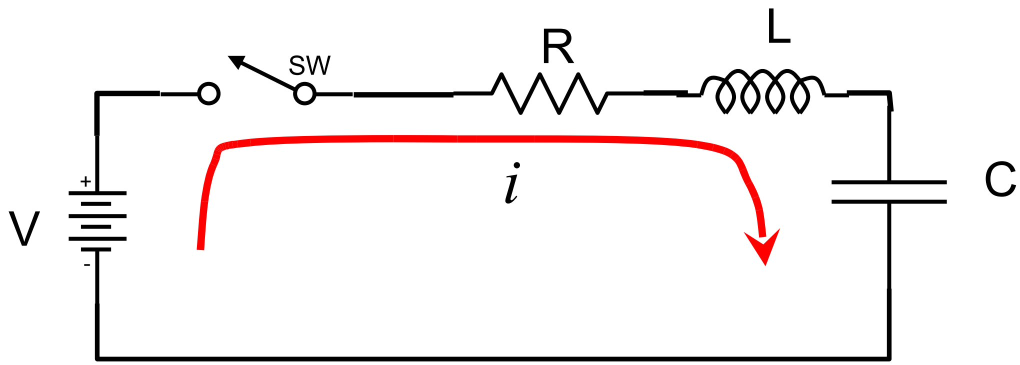
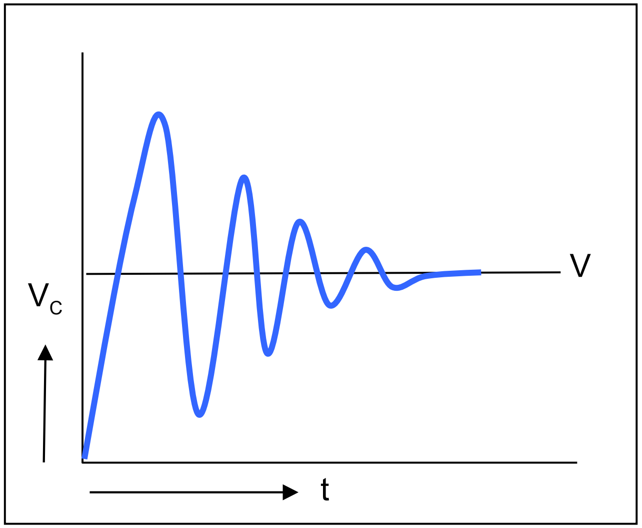
Transient Analysis: Series RLC Circuit



Current in an RLC circuit like shown
Is governed by the equation

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

We will analyse the situations with and without The source (V). The stored energy in C or L will force the current



Once the switch (SW) is closed, after some oscillatory period, current And voltage will settle. In steady state, Capacitor voltage (V_C) will approach V

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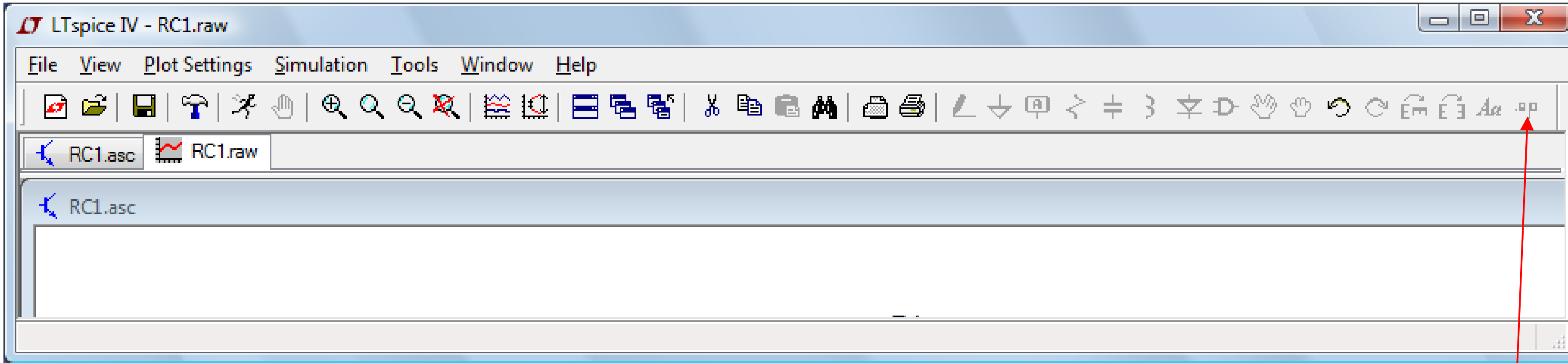
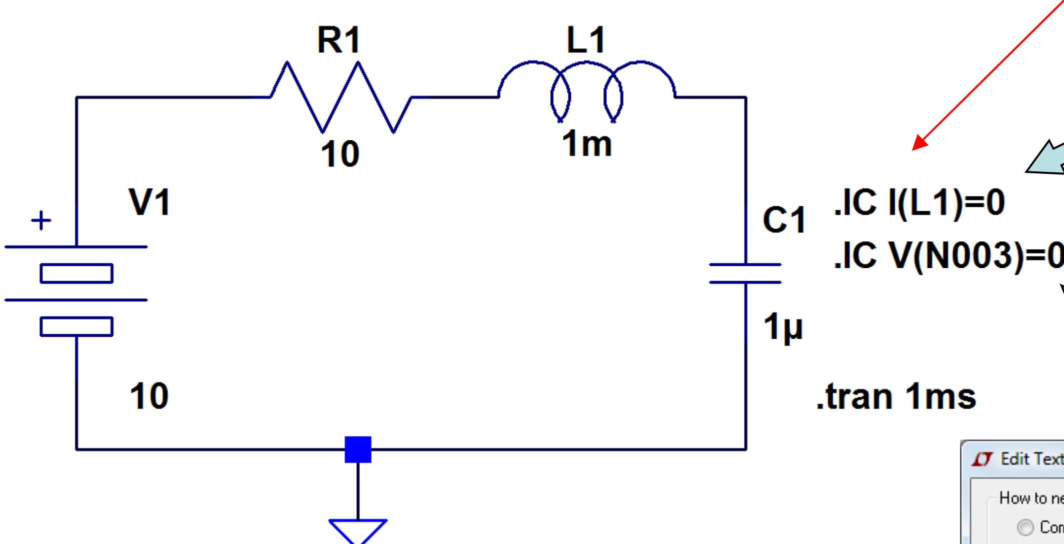
LT SPICE Simulation: Adding components

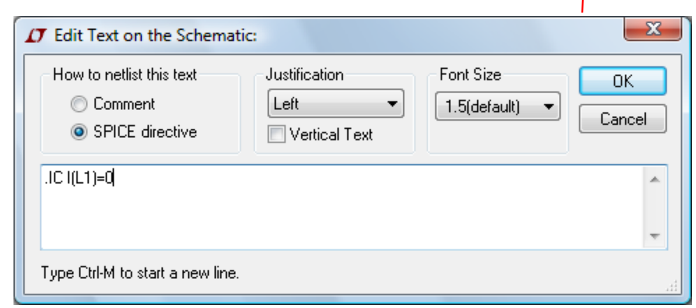


After adding the component and components values , add the **SPICE DIRECTIVE**

Considering there is no stored energy in the inductor (L) or Capacitor, C



At time, t=0, I =0
At time t =0, V_C=0



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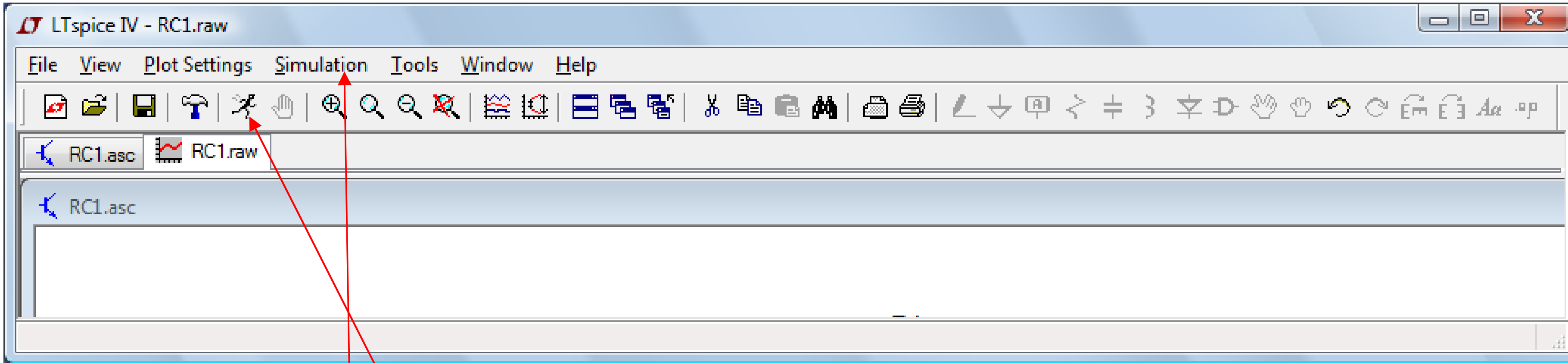
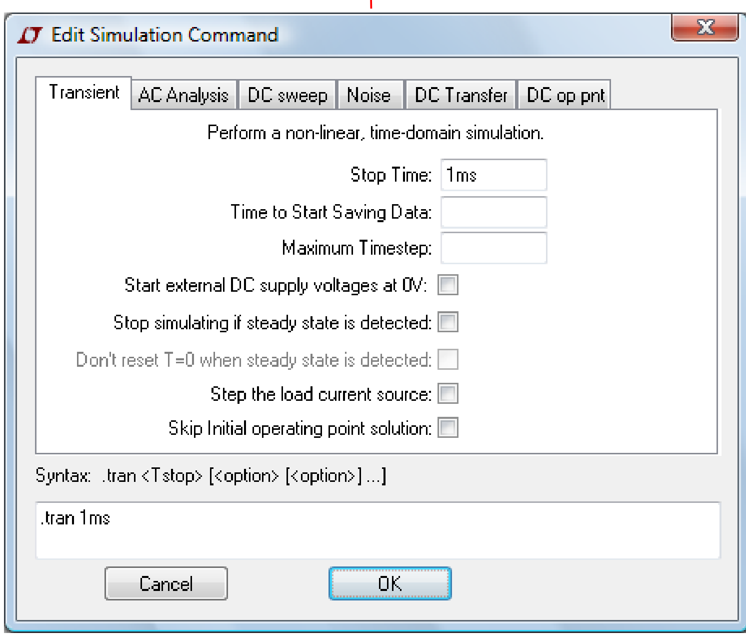
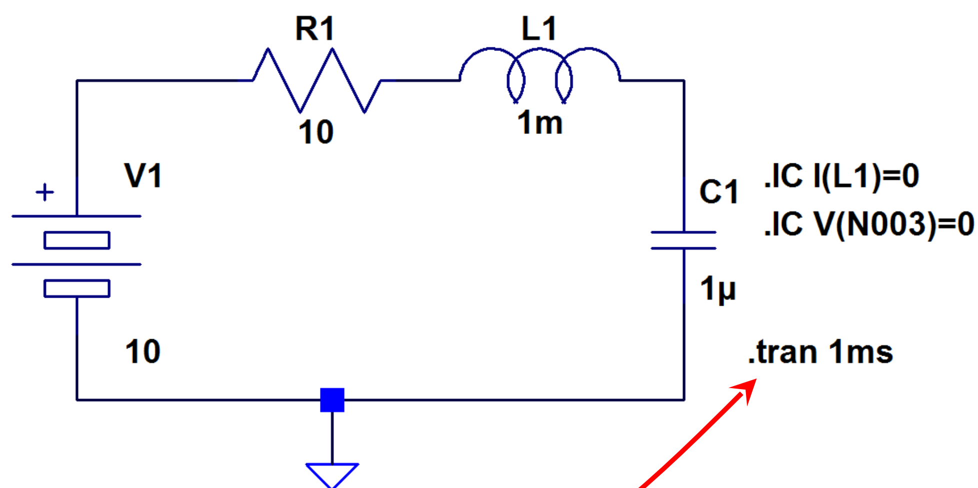
Run: Simulation



1. Simulation> Edit Simulation
2. RUN





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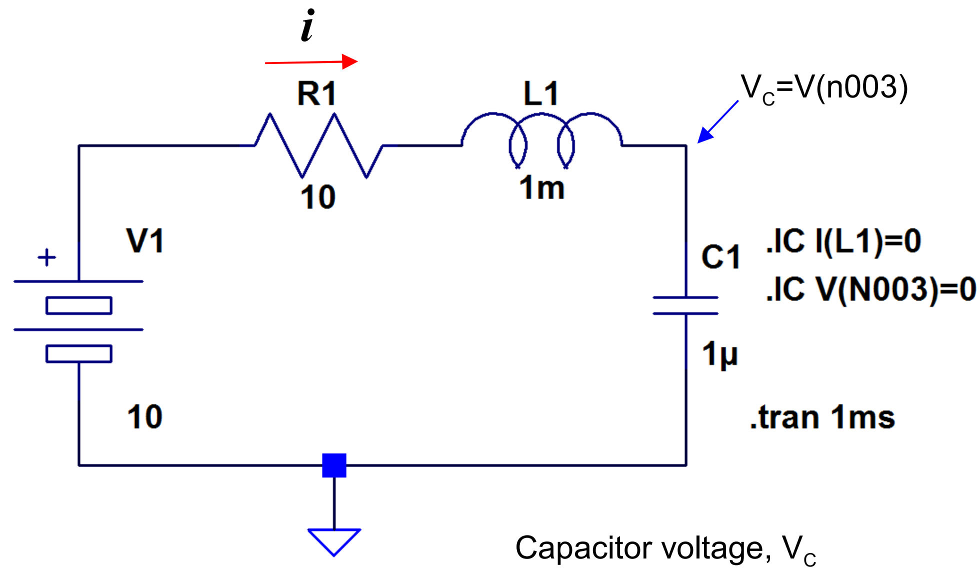
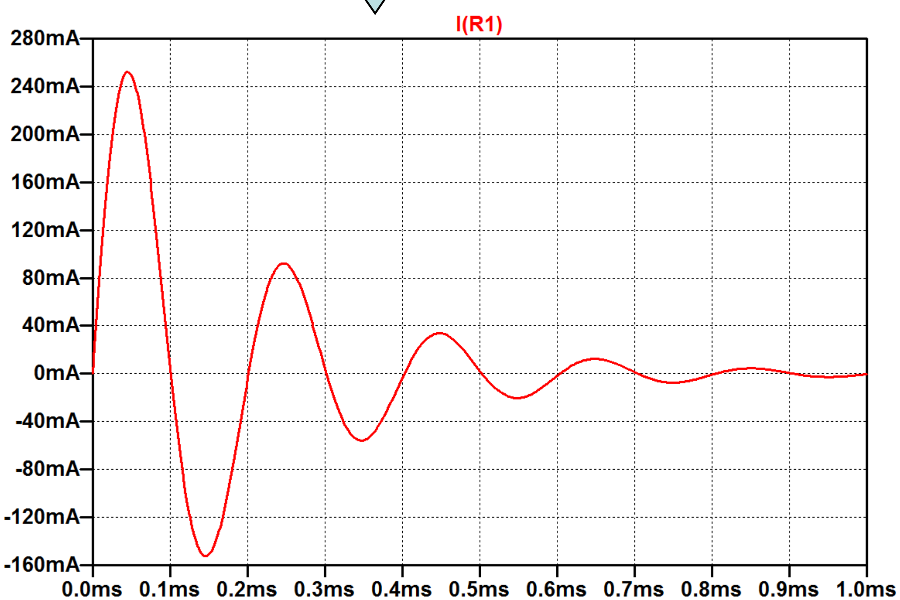
Run: Simulation

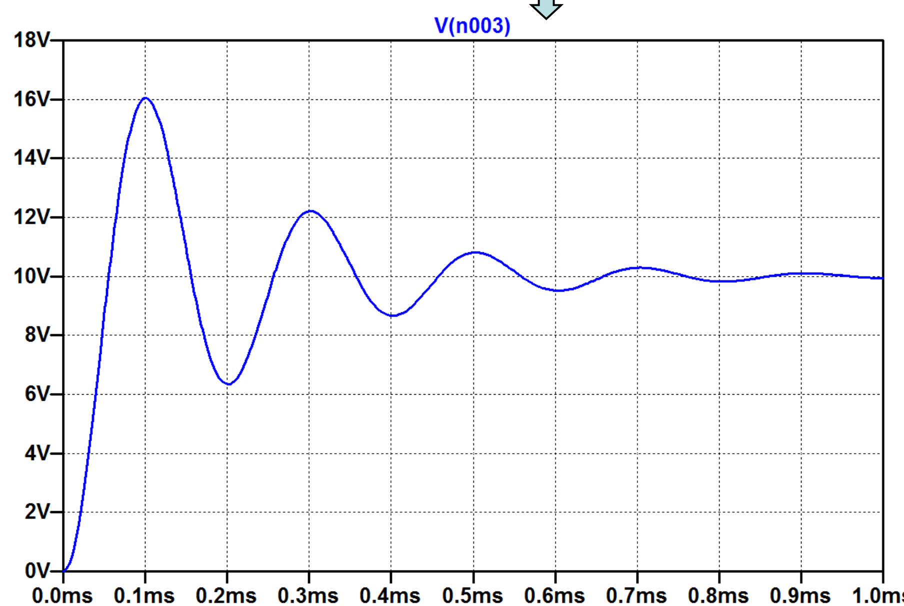
Running the simulation and placing the voltage probe at Node, n003 and clicking we find capacitor voltage and clicking the current probe either on R or L, we find current i



Current i



Capacitor voltage, V_C



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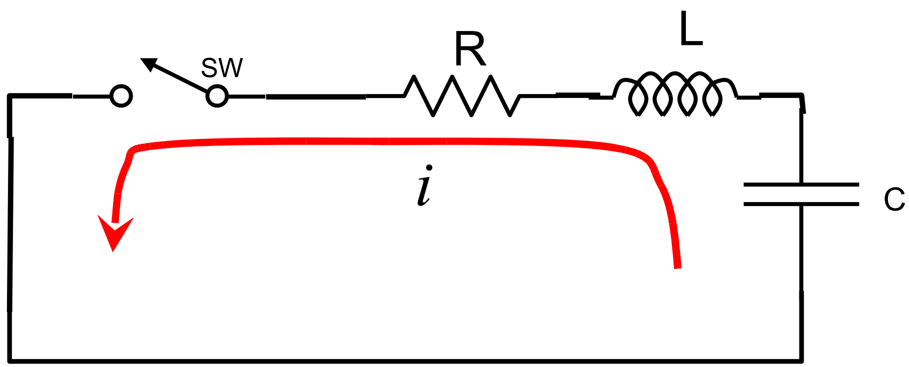
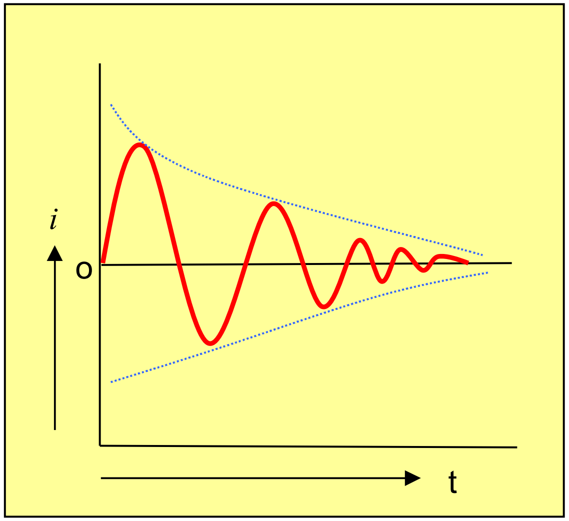
Stored Energy in Capacitor (C): No Power Source



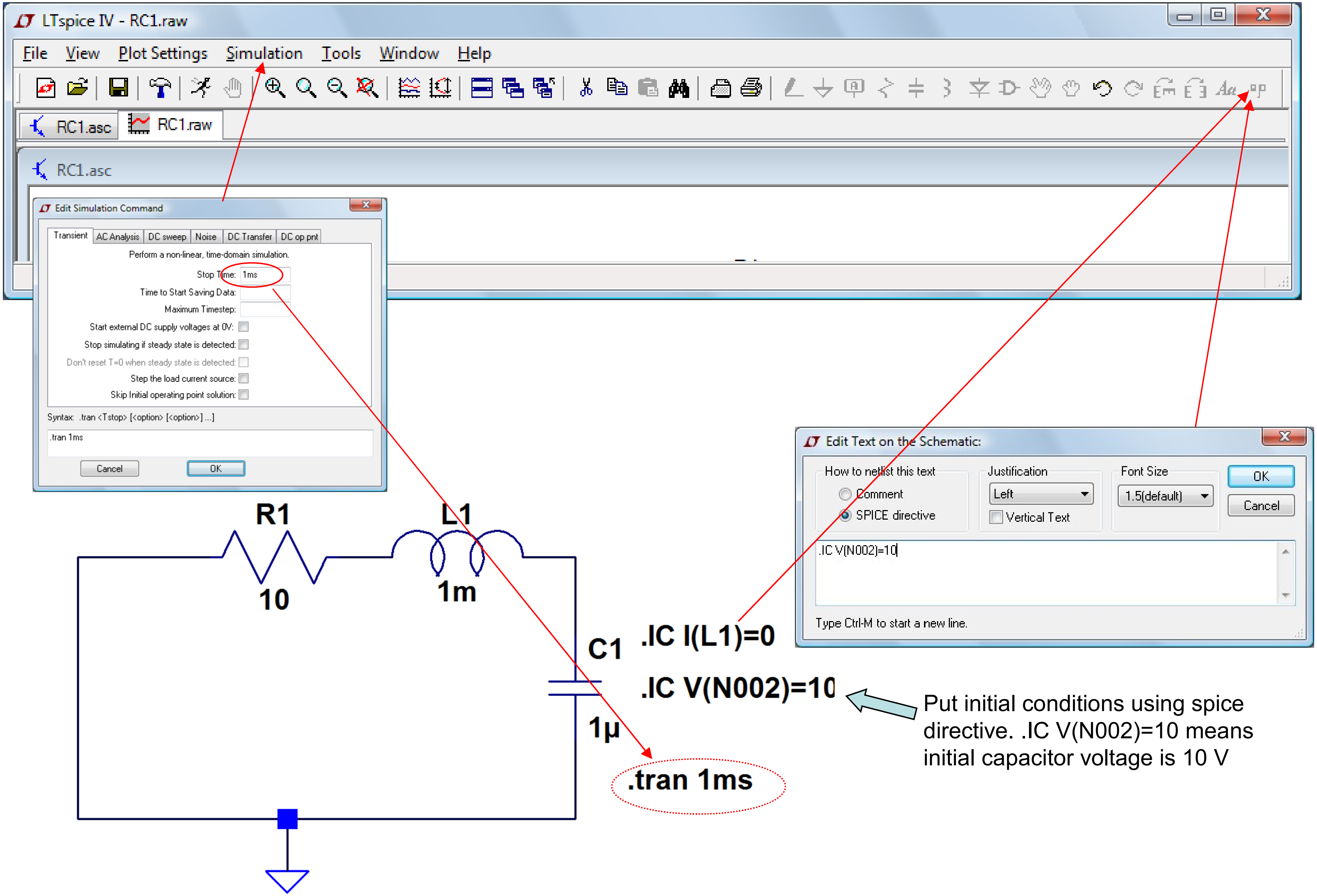
When the capacitor is charged and connected as shown, energy will be exchanged back and forth in-between the inductor and capacitor. However the resistor will start dissipating the energy. The resulting current is governed by the equation

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$



Simulation: Stored Capacitor in the Capacitor (C): No Power Source



Put initial conditions using spice directive. .IC V(N002)=10 means initial capacitor voltage is 10 V

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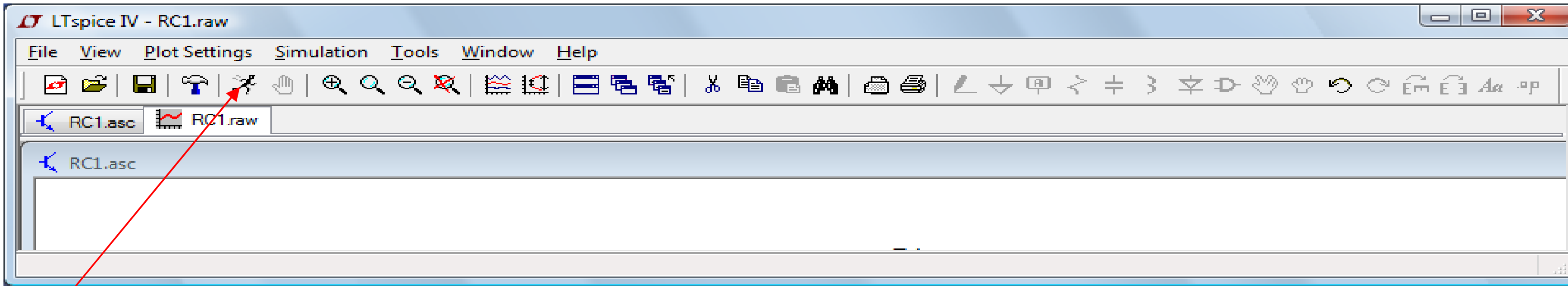
Run Simulation

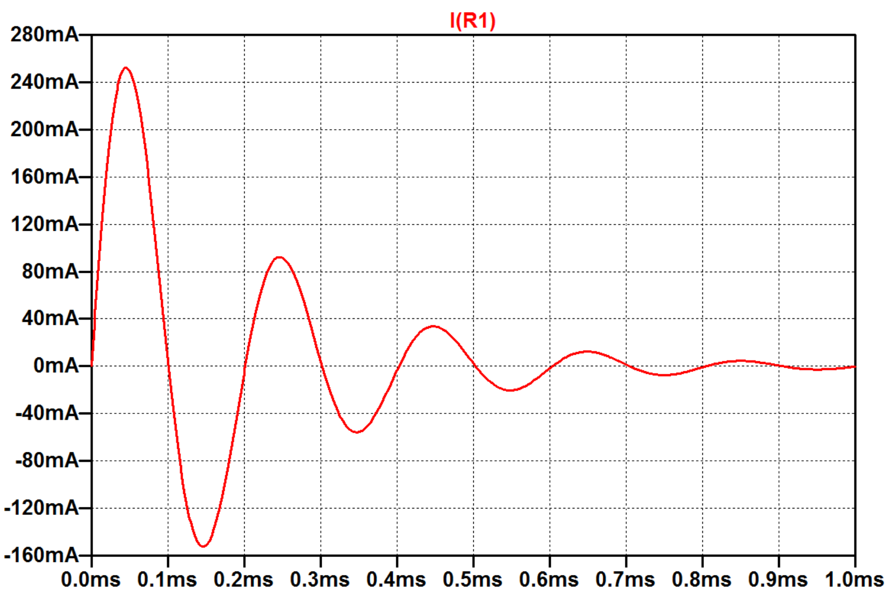


RUN

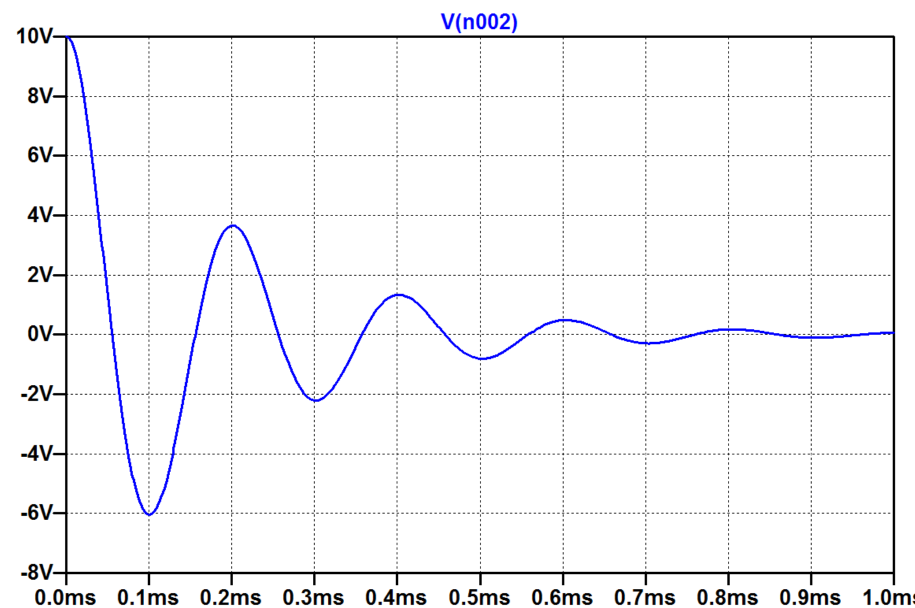
Running the simulation and placing the current Probe on either the resistor or the inductor, we find the oscillatory current as shown

Placing the voltage probe at node:N002 and Clicking we find the capacitor voltage waveform

Current



Capacitor voltage



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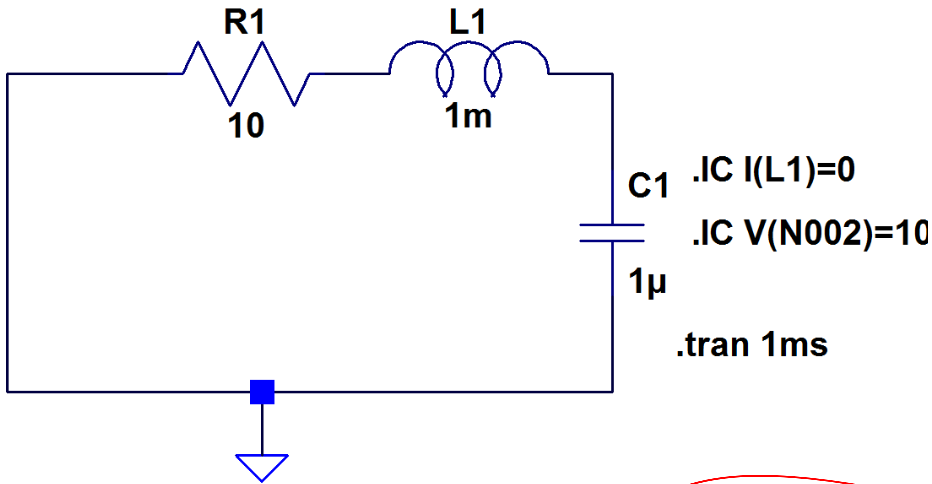
LTSpice to find Power and energy

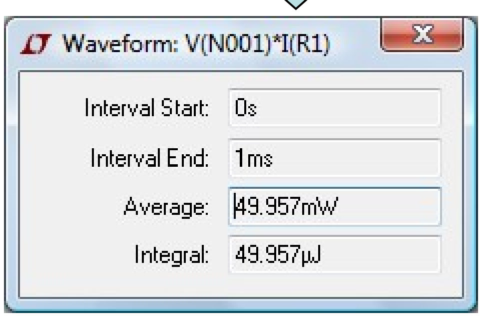


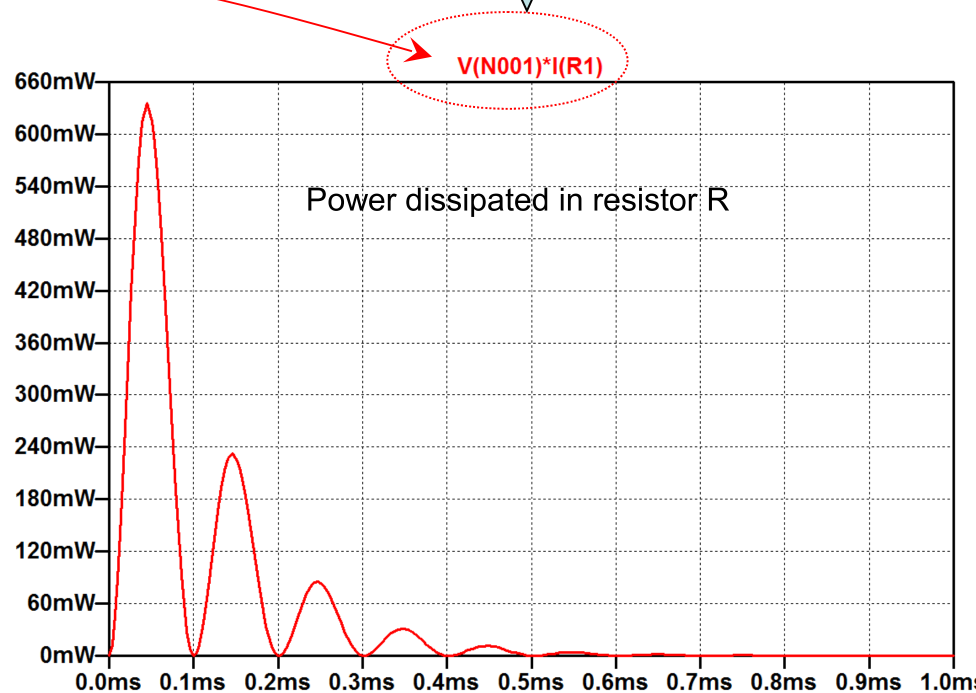
Press down **ALT** and put the cursor on R1 (You will See a thermometer icon) and click

You will get the power data as shown

Press down CTRL and place the cursor on V(N001)*R1 as shown and click

You will get the window like this





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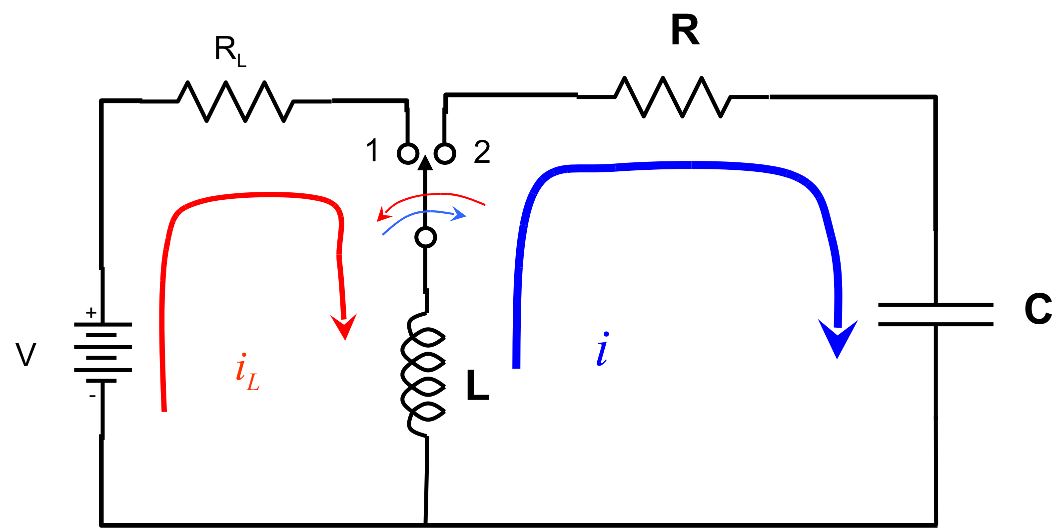
Stored Energy in Inductor (L): No Power Source

When the switch in position 1, maximum current V/R_L reaches in steady state. Now the switch is placed in position 2, the stored energy in the inductor will cause the current to oscillate in the LRC circuit.



In practice: Whenever the switch is about to release from position 1, there will be abrupt change in current, causes a high voltage to develop governed by the equation: $V_L = L\,di/dt$. The stored energy in the inductor ($1/2LI^2$) will be lost at the switch junction (1) (high voltage> Ionization>arching (heat)). However by electronic devices it is possible to release inductor-energy into an LRC circuit. I hope to discuss it later

However Let us consider an idealised situation that when the switch is in position 2 the energy ($1/2LI^2$) is released in the LRC circuit

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

Nature of current can be expressed by the equation

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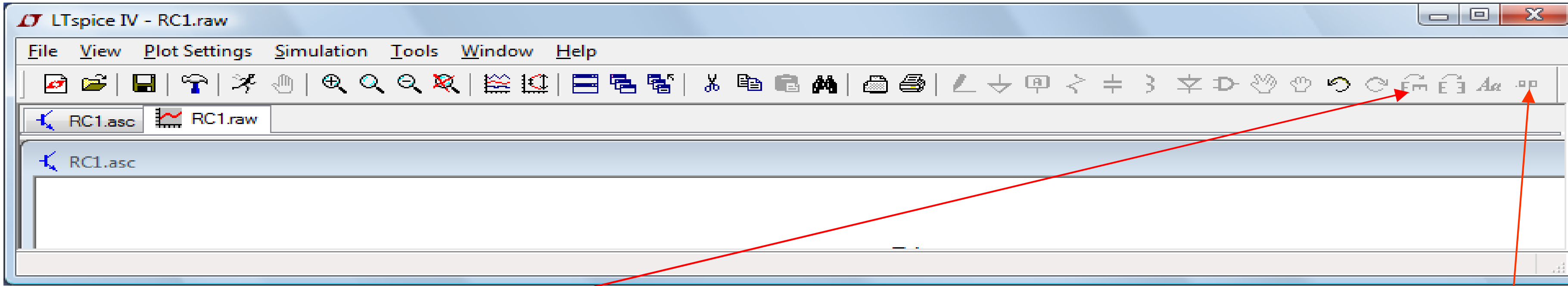
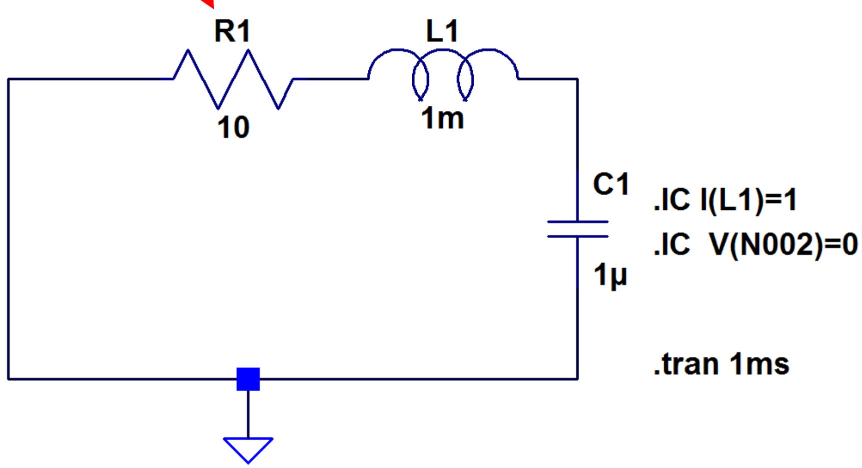
LT SPICE Simulation: Adding components

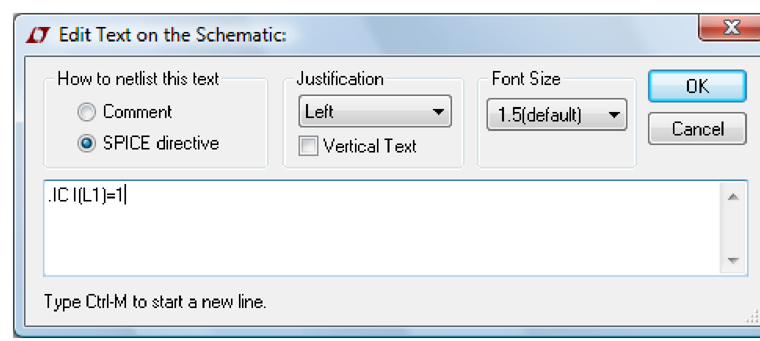


Rotate resistor R1 twice, which will give you the current in positive direction

Now initial inductor current is 1 Amp. and the capacitor voltage is 0 V



Set the spice directive



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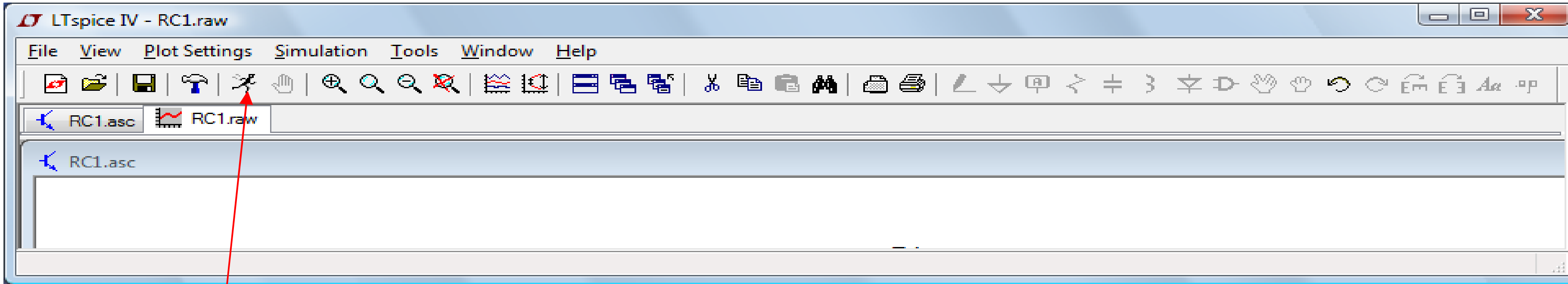
RUN: Simulation

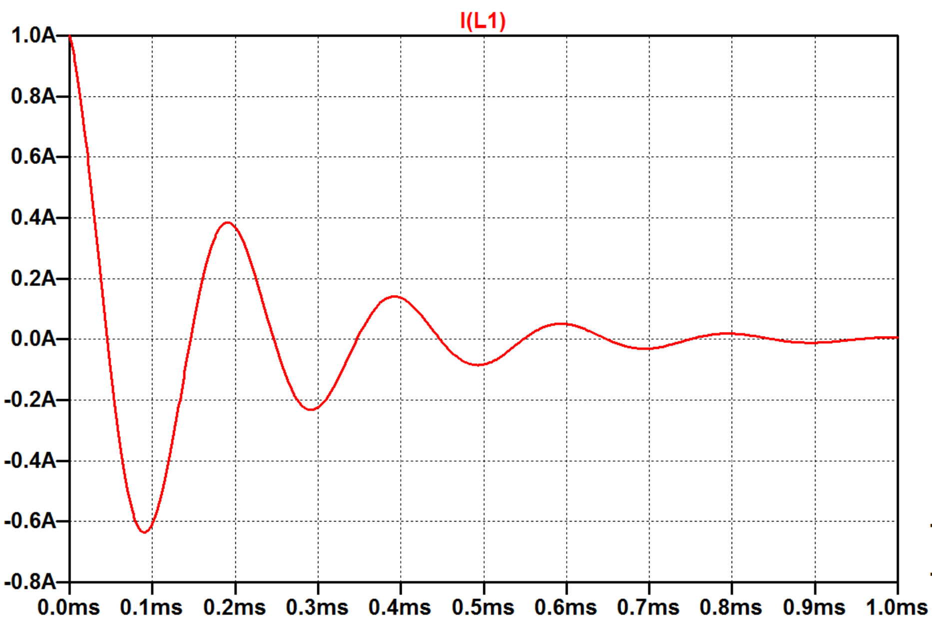


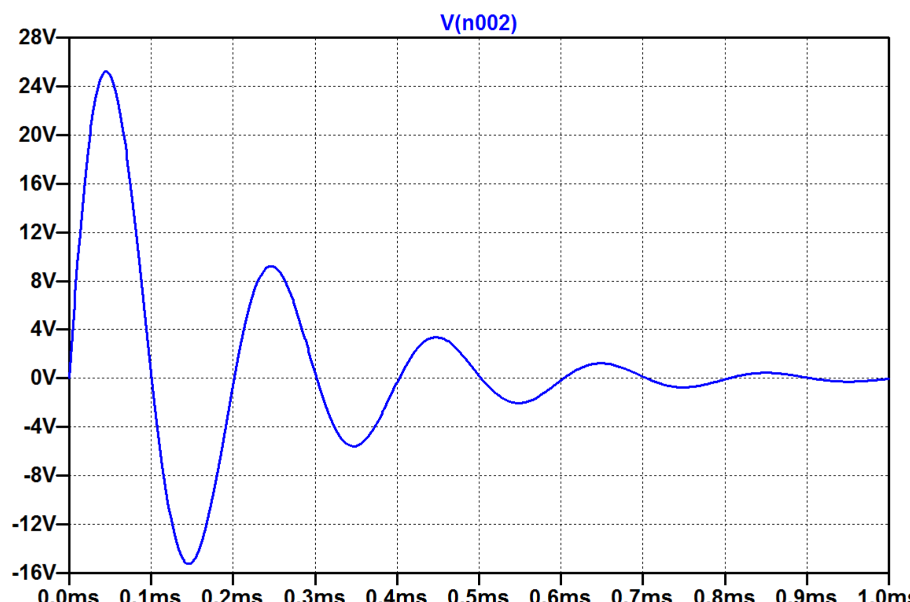
Run the simulation

We get current and capacitor voltage as shown:

Current

Capacitor voltage





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LTSpice to find Power and energy

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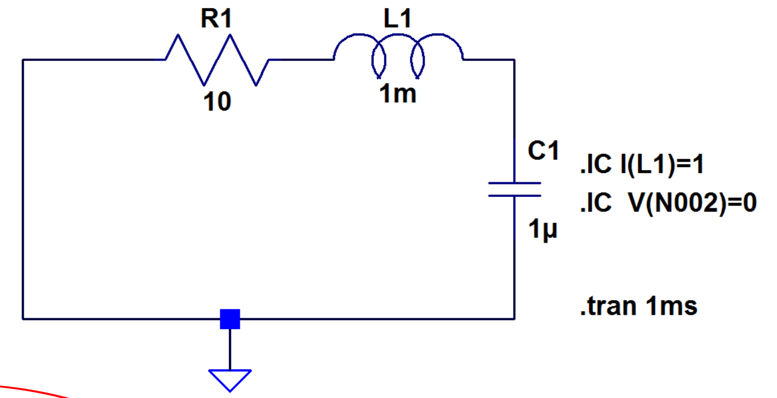
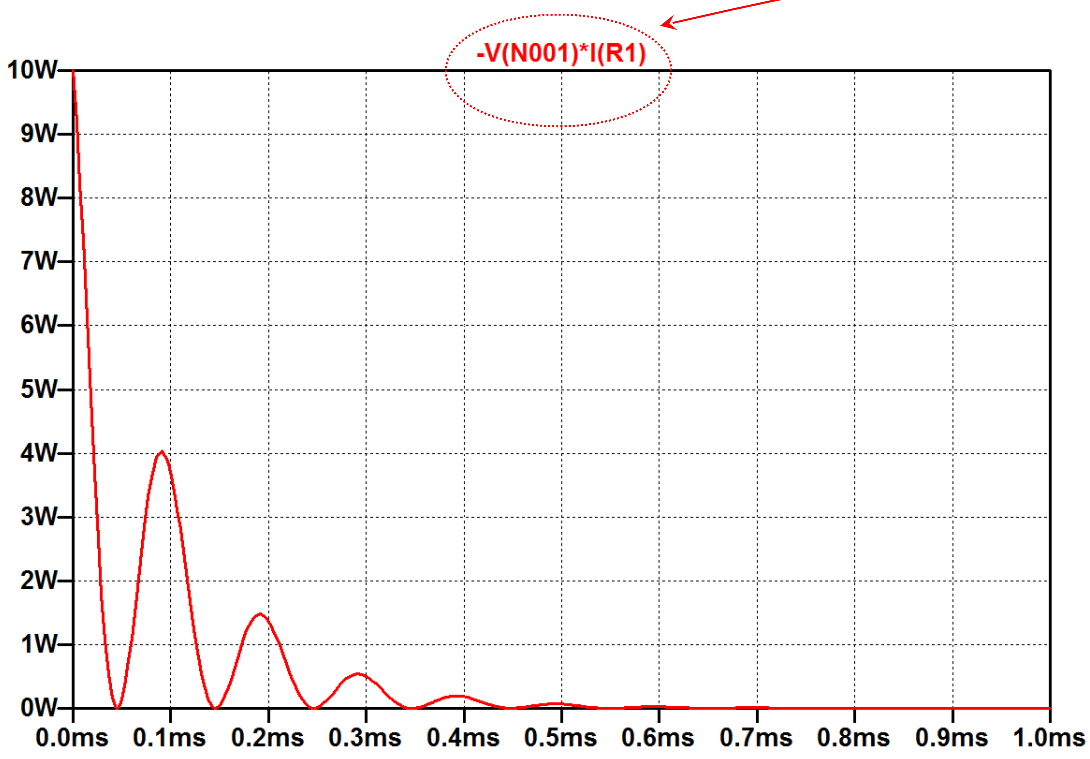
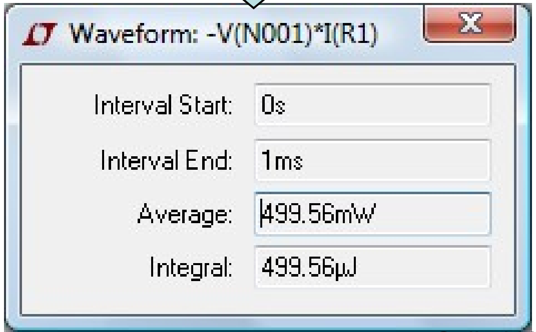




Press down **CTRL** and place the cursor on V(N001)*R1 as shown and click

You will get the window like this



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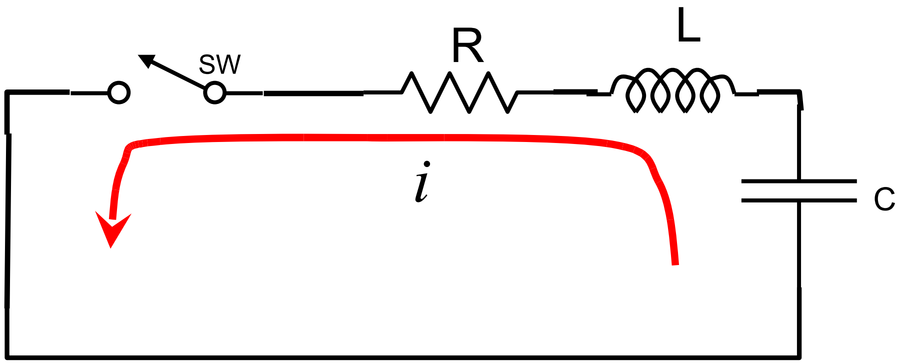
Under, over and Critically damped oscillation in LRC circuit



$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \qquad \boxed{1}$$

Let us consider: $i = Ae^{st}$ $\boxed{2}$

Putting equation 2 in 1:

$$LS^2Ae^{St} + RSAe^{St} + \frac{i}{C} = 0$$

$$Ae^{st}\left[LS^2 + RS + \frac{1}{C}\right] = 0$$

This is the characteristic equation
Which determines the circuit behaviour

Roots of this equation:

$$S_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$S_1 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$ ⟸ **Overdamped**

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$ ⟸ **Underdamped**

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$ ⟸ **Critically damped**

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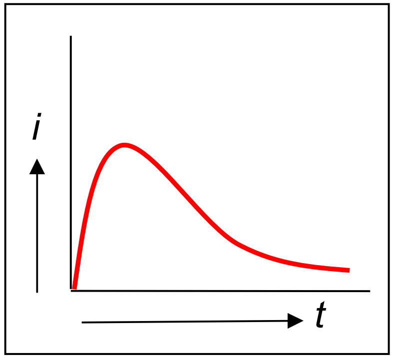
Under, over and Critically damped oscillation in LRC circuit

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

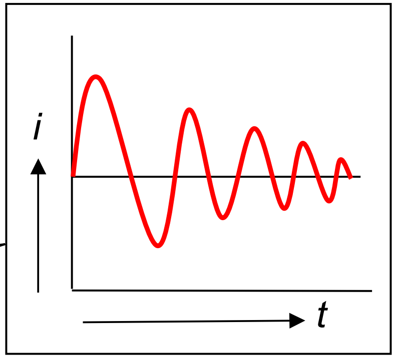
The solutions of the differential equation for these three conditions:

Overdamped $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ Let us consider, $\alpha = \frac{R}{2L}$ And $\omega_0 = \frac{1}{\sqrt{LC}}$



$$\alpha^2 > \omega_0^2$$

$$i(t) = Ae^{s_1 t} + Be^{s_2 t}$$ Where, $S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

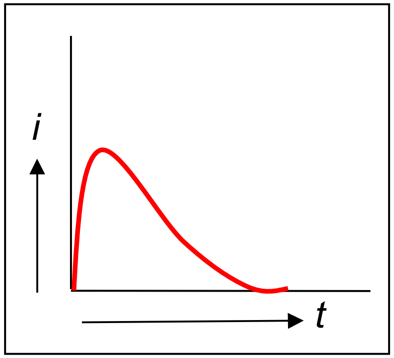


Underdamped $\alpha^2 < \omega_0^2$

$$i(t) = e^{-\alpha t}\left(A\cos\omega t + B\sin\omega t\right)$$ Where $\omega = \sqrt{\omega_o^2 - \alpha^2}$

Critically damped $\alpha^2 = \omega_0^2$

$$i(t) = \left(A + Bt\right)e^{-\alpha t}$$



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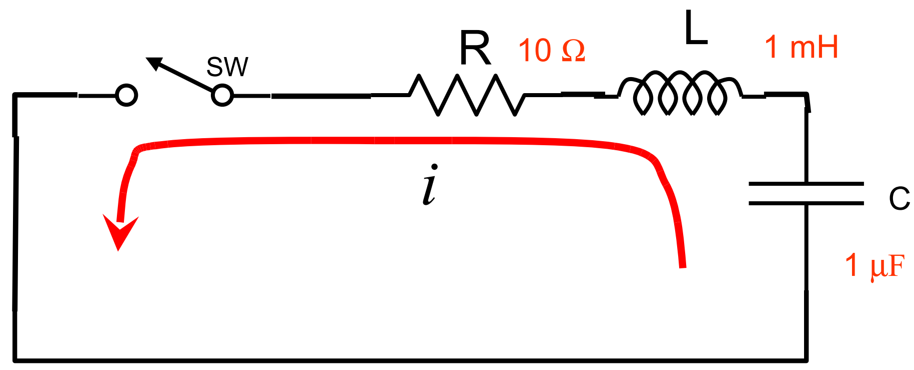
Simulation - Undedamped: LTSpice



$$\left(\frac{R}{2L}\right)^2 = 25\text{x}10^6$$

$$\frac{1}{LC} = 1\text{x}10^9$$

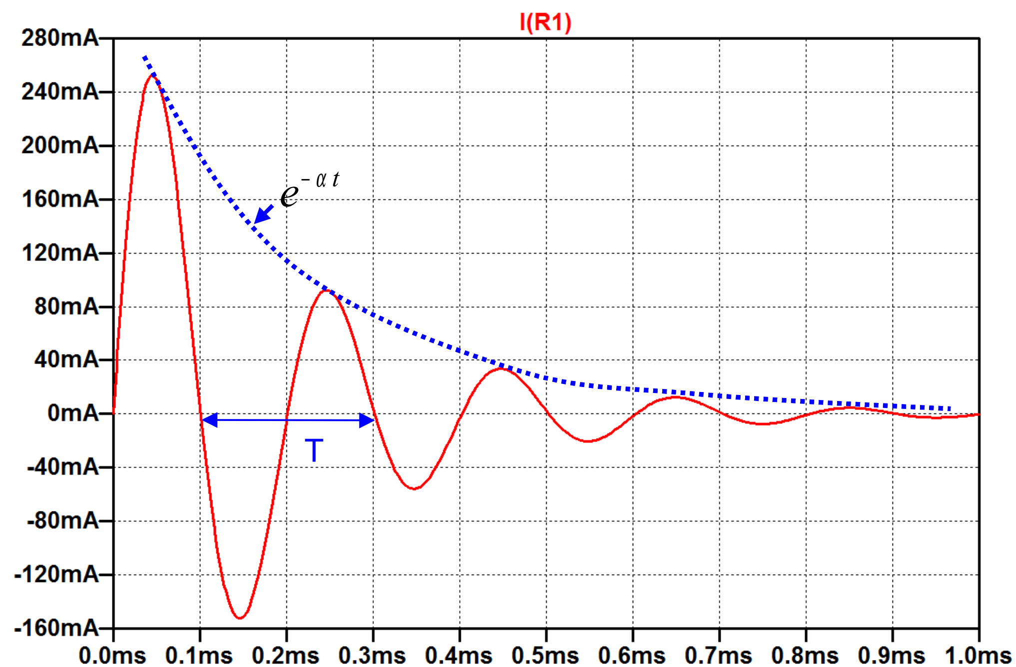
As, $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ It is a case of underdamped oscillation as we found before

$$i(t) = e^{-\alpha t}\left(A\cos\omega t + B\sin\omega t\right)$$

$$\omega = \sqrt{\omega_o^2 - \alpha^2}$$

=31224.99 Rad/sec

Time period, T = $\frac{2\pi}{\omega}$ = 201.2 μs



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Simulation - Overdamped: LTSpice

Let us consider, R=200 Ω

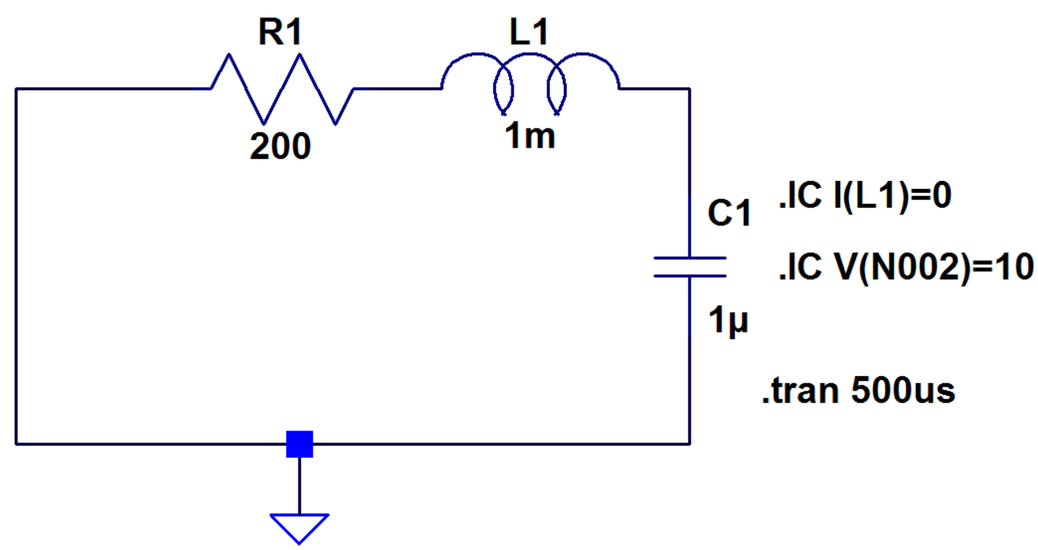
$$\left(\frac{R}{2L}\right)^2 = 1 \times 10^{10}$$

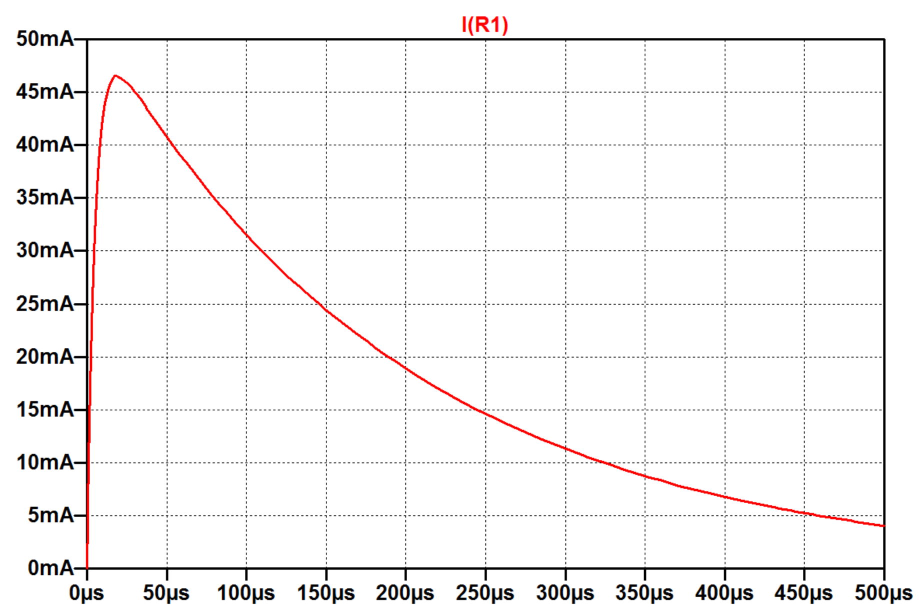
$$\frac{1}{LC} = 1\times10^{9}$$

$$\therefore \quad \left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

$$i(t) = Ae^{s_1 t} + Be^{s_2 t}$$

Running LTSpice simulation the same way as Beforewe find the overdamped behaviour as shown





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Simulation - Critically Damped: LTSpice

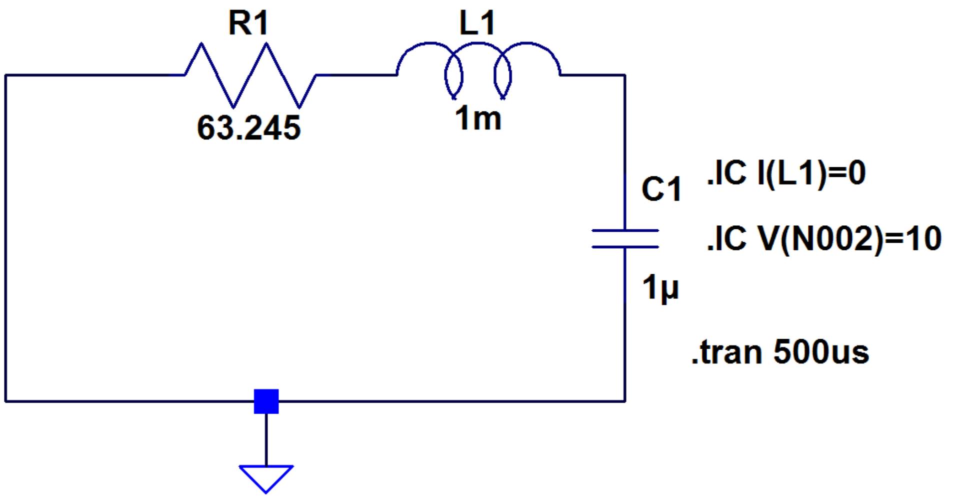
$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

$\alpha^2 = \omega_0^2$ Or $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

In our case with, L =1mH, C=1μF:

$R = 63.245\,\Omega$

$$i(t) = \left(A + Bt\right)e^{-\alpha t}$$



To find the time at which the current reaches the peak, we should differentiate i(t) and equate to 0:

$$\frac{di(t)}{di} = 0$$

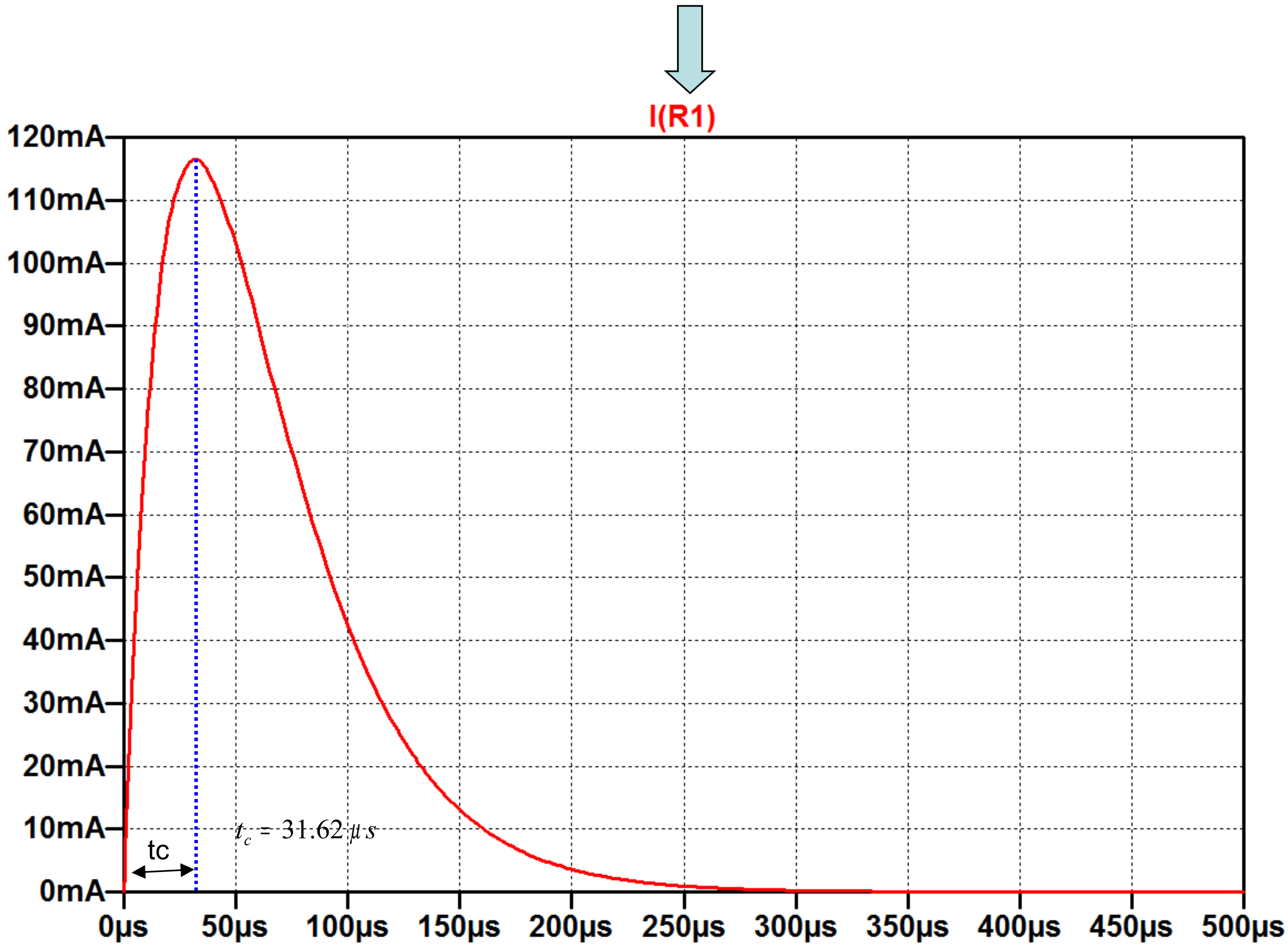
$$t_c = \frac{1}{\alpha} = \frac{2L}{R}$$

Putting $R = 63.245\,\Omega$

$$t_c = 31.62\,\mu s$$

Simulation - Critically Damped: LTSpice

Running the simulation gives us the current response, $i(t)$ as shown



The importance of critically damped circuit is, currently quickly reaches 0 without oscillating

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